Test 4
Applications of Integration and Discrete Random Variables

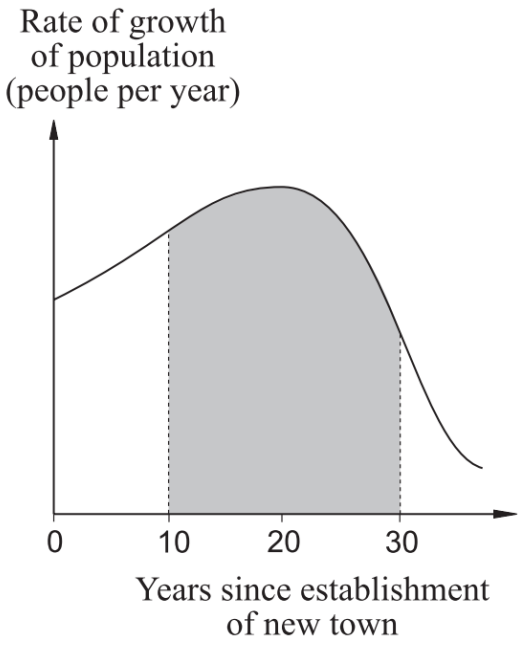
[This test contributes 6% towards the final year mark]

Name : _____ **Score :** _____ (out of 42)

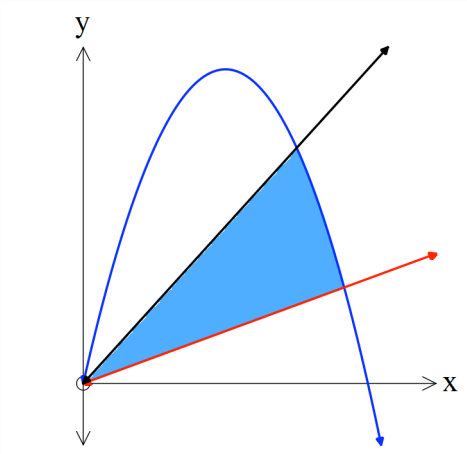
- 40 minutes are allocated for this task.
- CAS and/or scientific calculators are permitted.
- No notes of ANY nature are permitted.
- **Full marks may not be awarded to correct answers unless sufficient justification is given.**
- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**

Do NOT turn over this page until you are instructed to do so.

1. [2 marks]
   Describe, in words, what quantity is represented by the shaded area in the graph below.



2. [5 marks]
The graph below shows the sketch of the curve $y = 2x(6-x)$ and the lines $y = x$ and $y = 3x$.



Determine the **exact** area of the shaded region.

3. [3 + 2 + 2 = 7 marks]
A petrol tank, when full, contains 36 litres of petrol. It develops a small hole which widens as time goes by. The rate at which fuel leaks out (in litres per day) is given by the expression:

$$0.009t^2 + 0.08t + 0.01$$

where t is the time in days. When $t = 0$ the tank is full.

(a) Determine a formula for the amount of fuel lost after t days.

(b) How many litres of fuel does the tank lose on the tenth day?

(c) How much fuel is left in the tank after 15 days?

4. [4 + 2 + 2 = 8 marks]
An object is thrown vertically upward from a point O (at ground level) with velocity 49 ms^{-1}. The acceleration due to gravity is 9.8 ms^{-2} towards the centre of the Earth.

Determine:

(a) the height above O at any time t,

(b) the time(s), correct to 3 decimal places, the object is 15 metres above the ground,

(c) the maximum height reached.

5. [3 + 4 + 3 = 10 marks]
The discrete random variable X can only take the values 0, 1, 2, 3, 4, 5. The probability distribution of X is given by the following:

$P(X=0) = P(X=1) = P(X=2) = a$
$P(X=3) = P(X=4) = P(X=5) = b$ where a and b are constants.
$P(X \geq 2) = 3P(X < 2)$

(a) Determine the values of a and b.

(b) Show that the expectation of X is $\frac{23}{8}$ and determine the exact variance of X.

(c) Determine the exact probability that the sum of two independent observations from this distribution exceeds 7.

6. [3 + 2 + 2 + 3 = 10 marks]
On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw.

If the total score is 12, the statistician is paid \$6 by the gambler. If the total score is 8, the statistician is paid \$3 by the gambler. However, if both or either dice show a 1, the statistician pays the gambler \$2. Otherwise, no money changes hands.

Let \$ X be the amount paid to the statistician by the gambler.

(a) Complete the table below.

x		0	3	6
$P(X = x)$				

(b) Explain why the table in part (a) describes a probability distribution for the discrete random variable X.

(c) Show that, if the statistician played the game 100 times, his expected loss would be \$2.78, to the nearest cent.

(d) Find the amount, \$ a, that the \$6 would have to be changed to in order to make the game unbiased.

End of test

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